



## 2018-2019 Guide

April 1- April 18

### Eureka

#### Module 6: *Collecting and Displaying Data*



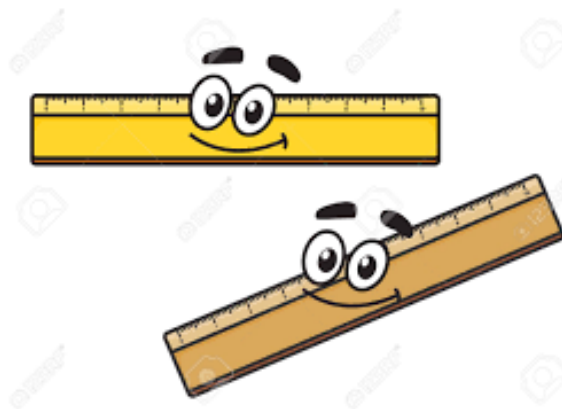
ORANGE PUBLIC SCHOOLS  
OFFICE OF CURRICULUM AND INSTRUCTION  
OFFICE OF MATHEMATICS

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## Module 6 Performance Overview

- Topic A begins with a lesson in which students generate categorical data, organize it, and then represent it in a variety of forms. By the end of the lesson, they show data in tape diagrams where units are equal groups with a value greater than 1.
- Students understand picture and bar graphs as vertical representations of tape diagrams, and apply well-practiced skip-counting and multiplication strategies to analyze them. Through problem solving, opportunities naturally surface for students to make observations, analyze, and answer questions such as, "How many more?" or "How many less?"
- In Topic B, students learn that intervals do not have to be whole numbers, but can also have fractional values that facilitate recording measurement data with greater precision. In Lesson 5, they generate a six-inch ruler marked in whole-inch, half-inch, and quarter-inch increments, using the Module 5 concept of partitioning a whole into parts. This creates a conceptual link between measurement and recent learning about fractions.



Module 6: Collecting and Displaying Data

<b><u>Pacing:</u></b> April 1- April 29 <b>9 Days</b>		
<b>Topic</b>	<b>Lesson</b>	<b>Lesson Objective/ Supportive Videos</b>
<b>Topic A:</b> Generate and Analyze Categorical Data	Lesson 1	Generate and organize data. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 2	Rotate tape diagrams vertically. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 3	Create scaled bar graphs. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 4	Solve one- and two-step problems involving graphs. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
<b>Topic B:</b> Generate and Analyze Measurement Data	Lesson 5	Create ruler with 1-inch, 1/2-inch, and 1/4-inch intervals and generate measurement data. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 6	Interpret measurement data from various line plots. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 7	Represent measurement data with line plots. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 8	Represent measurement data with line plots. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
<b>End Of Module Assessment</b>  <b>April 29, 2019</b>		

## NJSLS Standards:

### 3.MD.3

Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

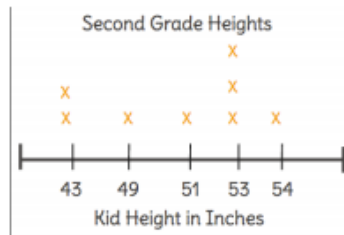
- Reading a graph requires students to interpret the information both horizontally and vertically.
- Pictures and bars can represent numbers in graphs.
- Modeling and promoting of the following vocabulary terms is crucial: *scale, scaled picture graph, scaled bar graph, line plot, key* and *data*.
- The way that data is collected, organized and displayed influences interpretation. Although intervals are not always in single units, students may count each square as one unit.
- While exploring data concepts, students should collect data, analyze data, and interpret data. Students should analyze, interpret and create bar graphs and pictographs in real world situations.



3.MD.4

Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

- Show measurements on a line plot to display the information in an organized way.



- Assure that students are accurately lining up the objects to be measure on the line plot and that the X's used are the same size to avoid misinterpretation of the data.
- Measure length using rulers marked with inch, quarter inch and half inch. Accurately measure several small objects using a standard ruler and display findings on a line plot. Third graders need many opportunities measuring the length of various objects in their environment.

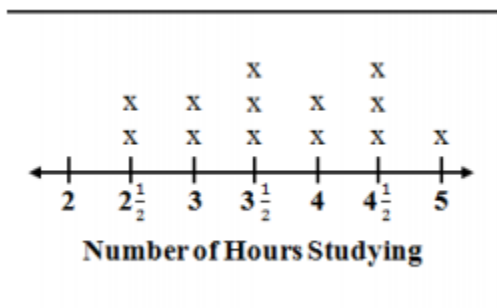
**Example:**

Measure objects in your desk to the nearest  $\frac{1}{2}$  inch or  $\frac{1}{4}$  of an inch.

Display data collected on a line plot.

How many objects measured  $\frac{1}{2}$  inch?  $\frac{1}{4}$  inch ?

Display data on line plots with horizontal scales in whole numbers, halves, and quarter.



- Students should connect their understanding of fractions to the measuring of one-half and one-quarter inch.

Common multiplication and division situations.<sup>1</sup>

	<b>UNKNOWN PRODUCT</b>	<b>GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)</b>	<b>NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)</b>
	<b><math>3 \times 6 = ?</math></b>	<b><math>3 \times ? = 18</math>, and <math>18 \div 3 = ?</math></b>	<b><math>? \times 6 = 18</math>, and <math>18 \div 6 = ?</math></b>
<b>EQUAL GROUPS</b>	There are 3 bags with 6 plums in each bag. How many plums are there in all? <i>Measurement example.</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? <i>Measurement example.</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? <i>Measurement example.</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
<b>ARRAYS<sup>2</sup>, AREA<sup>3</sup></b>	There are 3 rows of apples with 6 apples in each row. How many apples are there? <i>Area example.</i> What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
<b>COMPARE</b>	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? <i>Measurement example.</i> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? <i>Measurement example.</i> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? <i>Measurement example.</i> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
<b>GENERAL</b>	<b><math>a \times b = ?</math></b>	<b><math>a \times ? = p</math> and <math>p \div a = ?</math></b>	<b><math>? \times b = p</math>, and <math>p \div b = ?</math></b>

<sup>1</sup> The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

<sup>2</sup> Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

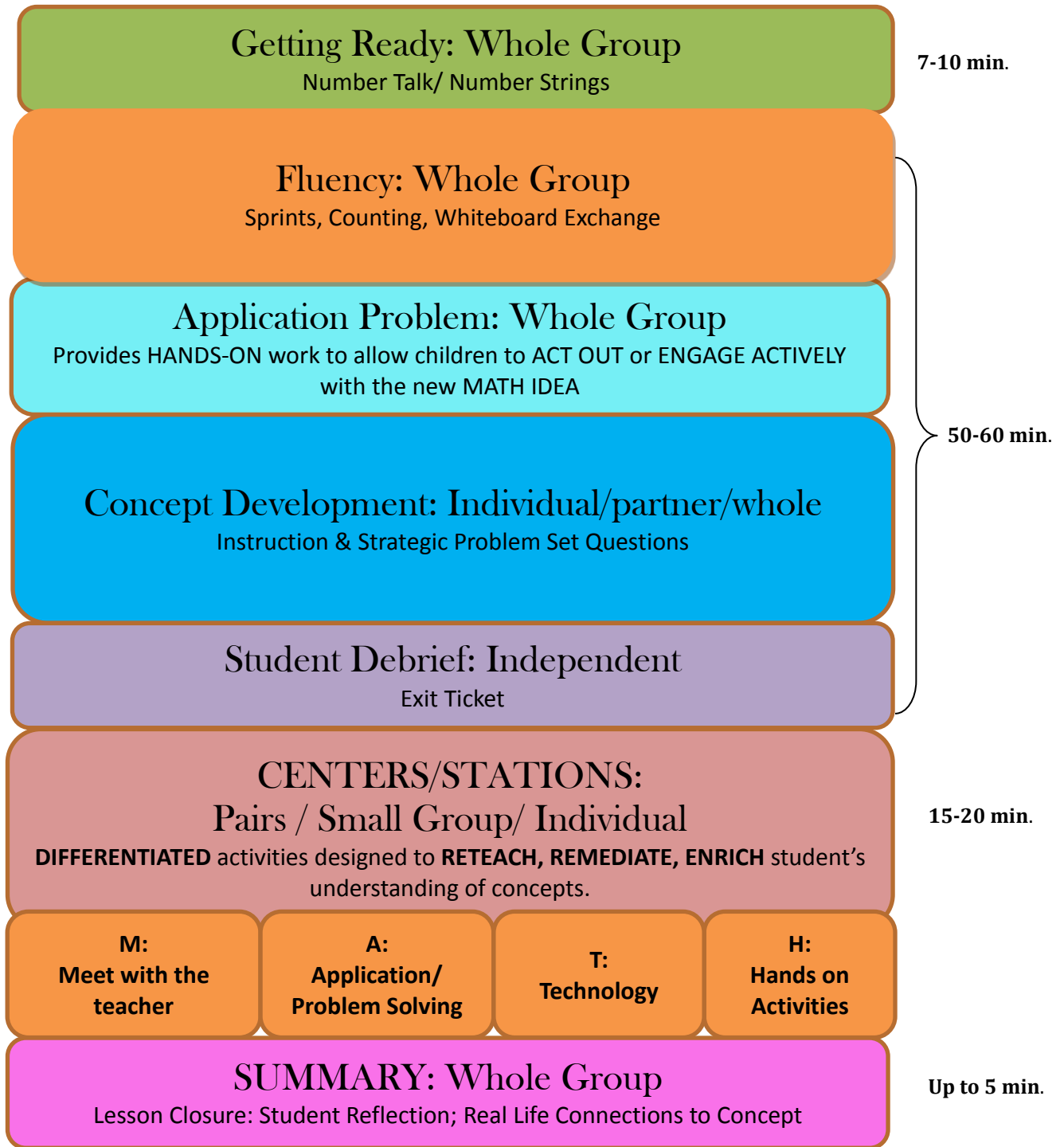
<sup>3</sup> The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

**Module 6 Assessment / Authentic Assessment Recommended Framework**

<b>Assessment</b>	<b>CCSS</b>	<b>Estimated Time</b>	<b>Format</b>
<i><b>Eureka Math Module 6:</b></i> <i>Collecting and Displaying Data</i>			
Authentic Assessment: Identifying a Fraction	3.MD.4	30 mins	Individual
Optional End of Module Assessment	3.MD.3 3.MD.4	1 Block	Individual



# Third Grade Ideal Math Block



## Eureka Lesson Structure:

### Fluency:

- Sprints
- Counting : Can start at numbers other than 0 or 1 and might include supportive concrete material or visual models
- Whiteboard Exchange

### Application Problem:

- Engage students in using the RDW Process
- Sequence problems from simple to complex and adjust based on students' responses
- Facilitate share and critique of various explanations, representations, and/or examples.

### Concept Development: (largest chunk of time)

#### Instruction:

- Maintain overall alignment with the objectives and suggested pacing and structure.
- Use of tools, precise mathematical language, and/or models
- Balance teacher talk with opportunities for peer share and/or collaboration
- Generate next steps by watching and listening for understanding

#### Problem Set: (Individual, partner, or group)

- Allow for independent practice and productive struggle
- Assign problems strategically to differentiate practice as needed
- Create and assign remedial sequences as needed

### Student Debrief:

- Elicit students thinking, prompt reflection, and promote metacognition through student centered discussion
- Culminate with students' verbal articulation of their learning for the day
- Close with completion of the daily Exit Ticket (opportunity for informal assessment that guides effective preparation of subsequent lessons) as needed.

## PARCC Assessment Evidence/Clarification Statements

CCSS	Evidence Statement	Clarification	MP
3.MD.3-1	<p>Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. For example, draw a bar graph in which each square in the bar graph might represent 5 pets</p>	<ul style="list-style-type: none"> <li>• Tasks involve no more than 10 items in 2-5 categories.</li> <li>• Categorical data should not take the form of a category that could be represented numerically (e.g. ages of students).</li> <li>• Tasks do not require students to create the entire graph, but might ask students to complete a graph or otherwise demonstrate knowledge of its creation.</li> </ul>	MP 2
3.MD.3-3	<p>Solve a put-together problem using information presented in a scaled bar graph, then use the result to answer a “how many more” or “how many less” problem using information presented in the scaled bar graph.</p> <p>Content Scope: 3.MD.3</p>	<ul style="list-style-type: none"> <li>• Tasks do not require computations beyond the grade 3 expectations.</li> </ul>	MP 4
3.MD.4	<p>Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.</p>		MP 2, 5

Student Name: \_\_\_\_\_

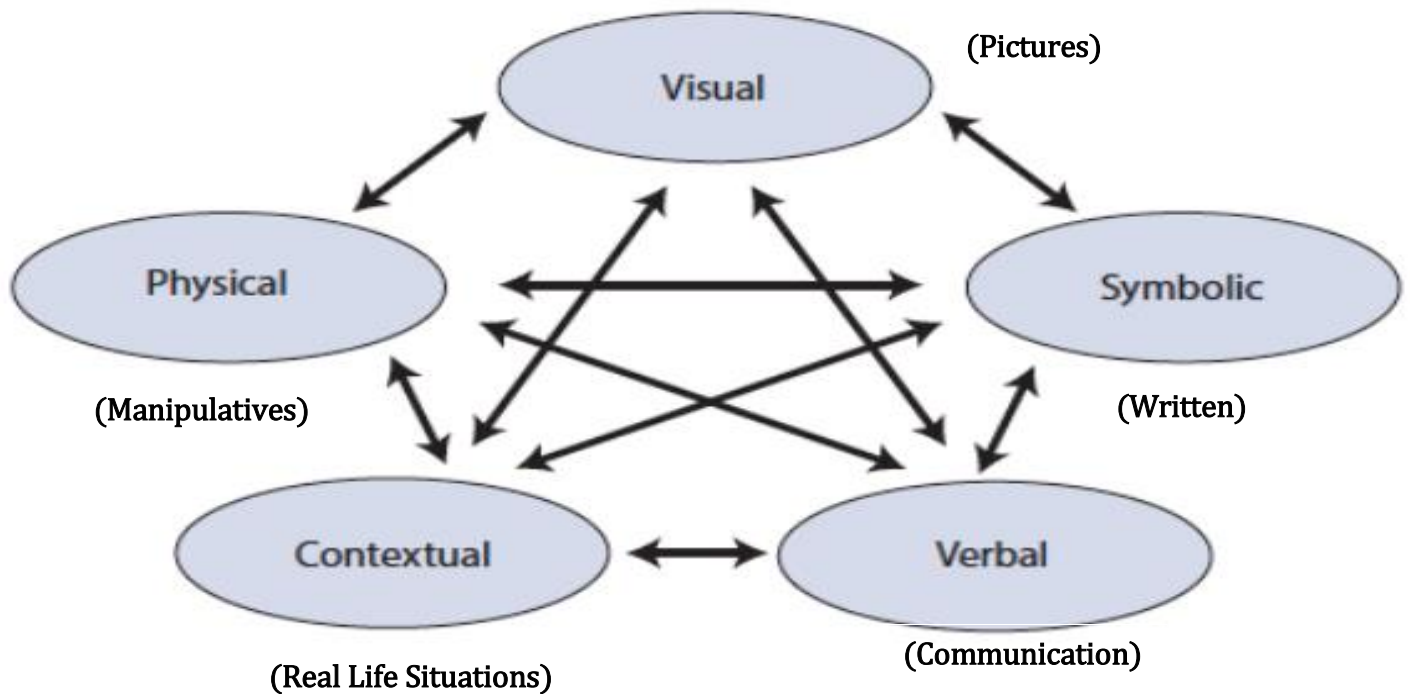
Task: \_\_\_\_\_

School: \_\_\_\_\_

Teacher: \_\_\_\_\_ Date: \_\_\_\_\_

"I CAN....."	STUDENT FRIENDLY RUBRIC				SCORE
	...a start 1	...getting there 2	...that's it 3	WOW! 4	
Understand	I need help.	I need some help.	I do not need help.	I can help a classmate.	
Solve	I am unable to use a strategy.	I can start to use a strategy.	I can solve it more than one way.	I can use more than one strategy and talk about how they get to the same answer.	
Say or Write	I am unable to say or write.	I can write or say some of what I did.	I can write and talk about what I did. I can write or talk about why I did it.	I can write and say what I did and why I did it.	
Draw or Show	I am not able to draw or show my thinking.	I can draw, but not show my thinking; or I can show but not draw my thinking;	I can draw and show my thinking	I can draw, show and talk about my thinking.	

## Use and Connection of Mathematical Representations



The Lesh Translation Model

Each oval in the model corresponds to one way to represent a mathematical idea.

**Visual:** When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

**Physical:** The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

**Verbal:** Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.

**Symbolic:** Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

**Contextual:** A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

### **The Lesh Translation Model: Importance of Connections**

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

## Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

**Concrete:** “Doing Stage”: Physical manipulation of objects to solve math problems.

**Pictorial:** “Seeing Stage”: Use of imaged to represent objects when solving math problems.

**Abstract:** “Symbolic Stage”: Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

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## Read, Draw, Write Process

**READ** the problem. Read it over and over.... And then read it again.

**DRAW** a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing?

**WRITE** your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

## Mathematical Discourse and Strategic Questioning

Discourse involves asking strategic questions that elicit from students their understanding of the context and actions taking place in a problem, how a problem is solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

### Teacher Questioning:

Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple "yes" or "no," or do they invite students to deepen their understanding?



The most  
important thing  
is to NEVER  
stop  
questioning

*Albert Einstein*



To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Dr. Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.

**100** questions that promote

# Mathematical Discourse

Help students **work together** to make sense of mathematics

- 1 What **strategy** did you use?
- 2 Do you **agree**?
- 3 Do you **disagree**?
- 4 Would you **ask the rest of the class** that question?
- 5 Could you **share your method** with the class?
- 6 What part of what he said **do you understand**?
- 7 Would someone like to **share** \_\_\_?
- 8 Can you **convince the rest of us** that your answer makes sense?
- 9 **What do others think** about what [student] said?
- 10 Can someone **retell or restate** [student]'s explanation?
- 11 Did you **work together**? In what way?
- 12 Would anyone like to **add to what was said**?
- 13 Have you **discussed** this with your group? With others?
- 14 Did anyone get a **different answer**?
- 15 **Where** would you go for **help**?
- 16 **Did everybody get a fair chance** to talk, use the manipulatives, or be the recorder?
- 17 How could you help another student **without telling them the answer**?
- 18 **How would you explain** \_\_\_ to someone who missed class today?

Help students **rely more on themselves** to determine whether something is mathematically correct

- 19 Is this a **reasonable answer**?
- 20 Does that make **sense**?
- 21 **Why** do you think that? Why is that true?
- 22 Can you **draw a picture or make a model** to show that?
- 23 **How** did you reach that conclusion?
- 24 Does anyone want to **revise** his or her answer?
- 25 **How were you sure** your answer was right?

Ready

## Help students learn to reason mathematically

- 26 How did you **begin** to think about this problem?
- 27 What is **another way** you could solve this problem?
- 28 How could you **prove** \_\_\_\_\_?
- 29 Can you **explain how your answer is different from or the same as** [student]'s answer?
- 30 Let's **break the problem into parts**. What would the parts be?
- 31 Can you **explain this part more specifically**?
- 32 Does that **always work**?
- 33 Can you think of a case where that **wouldn't work**?
- 34 How did you **organize** your information? Your thinking?

## Help students with problem comprehension

- 39 What is this problem about? What can you **tell me about it**?
- 40 Do you need to **define or set limits** for the problem?
- 41 How would you **interpret** that?
- 42 Could you **reword that in simpler terms**?
- 43 Is there something that can be **eliminated** or that is **missing**?
- 44 Could you **explain** what the problem is asking?
- 45 What **assumptions** do you have to make?
- 46 What do you **know** about this part?
- 47 Which words were **most important**? Why?

## Help students evaluate their own processes and engage in productive peer interaction

- 35 What do you need to do **next**?
- 36 What have you **accomplished**?
- 37 What are your **strengths and weaknesses**?
- 38 Was your **group participation appropriate and helpful**?



## Help students learn to **conjecture, invent, and solve** problems

- 48 What would happen if \_\_\_?
- 49 Do you see a **pattern**?
- 50 What are some **possibilities** here?
- 51 Where could you find the **information** you need?
- 52 How would you **check your steps** or your answer?
- 53 What **did not work**?
- 54 How is your solution method the **same as or different from** [student]'s method?
- 55 Other than retracing your steps, **how can you determine** if your answers are appropriate?
- 56 How did you **organize** the information? Do you have a **record**?
- 57 How could you solve this using **tables, lists, pictures, diagrams**, etc.?
- 58 What have you tried? What **steps** did you take?
- 59 How would it look if you used this **model** or these **materials**?
- 60 How would you draw a **diagram** or **make a sketch** to solve the problem?
- 61 Is there **another possible answer**? If so, explain.
- 62 Is there **another way to solve** the problem?
- 63 Is there **another model** you could use to solve the problem?
- 64 Is there anything you've **overlooked**?
- 65 **How did you think** about the problem?
- 66 What was your **estimate or prediction**?
- 67 How **confident** are you in your answer?
- 68 **What else** would you like to know?
- 69 What do you think comes **next**?
- 70 Is the solution **reasonable**, considering the context?
- 71 Did you have a **system**? Explain it.
- 72 Did you have a **strategy**? Explain it.
- 73 Did you have a **design**? Explain it.



## Help students learn to connect mathematics, its ideas, and its application

- 74 What is the **relationship** between \_\_\_ and \_\_\_?
- 75 Have we ever solved a problem **like this before**?
- 76 What uses of mathematics did you find in the **newspaper** last night?
- 77 What is the **same**?
- 78 What is **different**?
- 79 Did you use skills or build on concepts that were **not necessarily mathematical**?
- 80 Which **skills or concepts** did you use?
- 81 What **ideas** have we explored before that were useful in solving this problem?

- 82 Is there a **pattern**?
- 83 **Where else** would this strategy be useful?
- 84 How does this **relate** to \_\_\_?
- 85 Is there a **general rule**?
- 86 Is there a **real-life situation** where this could be used?
- 87 How would your method work with **other problems**?
- 88 What other problem does this seem to **lead to**?

### Help students persevere

- 95 What was **one thing you learned** (or two, or more)?
- 96 Did you **notice any patterns**? If so, describe them.
- 97 What **mathematics topics** were used in this investigation?
- 98 What were the **mathematical ideas** in this problem?
- 99 What is mathematically **different about these two situations**?
- 100 What are the **variables** in this problem? What stays **constant**?

- 89 Have you tried making a **guess**?
- 90 **What else** have you tried?
- 91 Would **another method** work as well or better?
- 92 Is there **another way** to draw, explain, or say that?
- 93 Give me another **related problem**. Is there an easier problem?
- 94 How would you **explain** what you know right now?

### Help students focus on the mathematics from activities

## **Conceptual Understanding**

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

## **Procedural Fluency**

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures.

## **Math Fact Fluency: Automaticity**

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the mind with the low-level details required, allowing it to become an automatic response pattern or habit. It is usually the result of learning, repetition, and practice.

### **3-5 Math Fact Fluency Expectation**

**3.OA.C.7:** Single-digit products and quotients (Products from memory by end of Grade 3)

**3.NBT.A.2:** Add/subtract within 1000

**4.NBT.B.4:** Add/subtract within 1,000,000/ Use of Standard Algorithm

**5.NBT.B.5:** Multi-digit multiplication/ Use of Standard Algorithm

## Evidence of Student Thinking

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

## Mathematical Proficiency

To be mathematically proficient, a student must have:

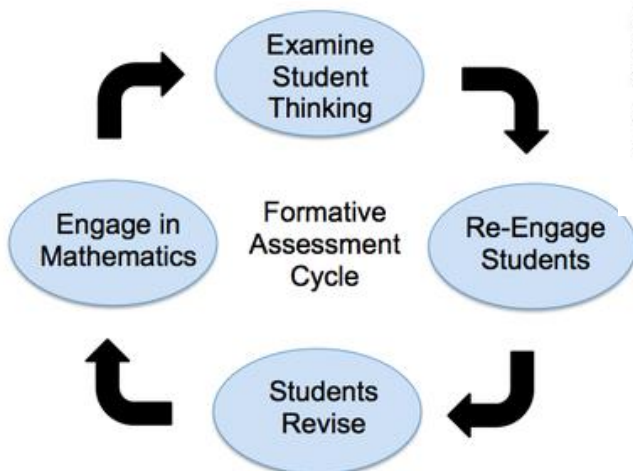
- Conceptual understanding: comprehension of mathematical concepts, operations, and relations;
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- Strategic competence: ability to formulate, represent, and solve mathematical problems;
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

## Evidence should:

- Provide a window in student thinking;
- Help teachers to determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.

*Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.*

(William 2007, pp. 1054; 1091)



## Connections to the Mathematical Practices

### **Student Friendly Connections to the Mathematical Practices**

1. I can solve problems without giving up.
2. I can think about numbers in many ways.
3. I can explain my thinking and try to understand others.
4. I can show my work in many ways.
5. I can use math tools and tell why I choose them.
6. I can work carefully and check my work.
7. I can use what I know to solve new problems.
8. I can discover and use short cuts.

### The Standards for Mathematical Practice:

Describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

<b>1</b>	<p><b>Make sense of problems and persevere in solving them</b></p> <p>In <b>third</b> grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” They listen to the strategies of others and will try approaches. They often will use another method to check their answers.</p>
<b>2</b>	<p><b>Reason abstractly and quantitatively</b></p> <p>In <b>third</b> grade, students should recognize that number represents a specific quantity. They connect quantity to written symbols and create logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities</p>
<b>3</b>	<p><b>Construct viable arguments and critique the reasoning of others</b></p> <p>In <b>third</b> grade, mathematically proficient students may construct viable arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like, “How did you get that?” and “Why is it true?” They explain their thinking to others and respond to others’ thinking.</p>
<b>4</b>	<p><b>Model with mathematics</b></p> <p>Mathematically proficient students experiment with representing problem situations in multiple ways including numbers, words (mathematical language) drawing pictures, using objects, acting out, making chart, list, or graph, creating equations etc...Students need opportunities to connect different representations and explain the connections. They should be able to use all of the representations as needed. <b>Third</b> graders should evaluate their results in the context of the situation and reflect whether the results make any sense.</p>
<b>5</b>	<p><b>Use appropriate tools strategically</b></p> <p><b>Third</b> graders should consider all the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For example, they might use graph paper to find all possible rectangles with the given perimeter. They compile all possibilities</p>

	into an organized list or a table, and determine whether they all have the possible rectangles.
<b>6</b>	<b>Attend to precision</b>
	Mathematical proficient <b>third</b> graders develop their mathematical communication skills; they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying their units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle the record their answer in square units.
<b>7</b>	<b>Look for and make use of structure</b>
	In <b>third</b> grade, students should look closely to discover a pattern of structure. For example, students properties of operations as strategies to multiply and divide. (commutative and distributive properties.
<b>8</b>	<b>Look for and express regularity in repeated reasoning</b>
	Mathematically proficient students in third <b>grade</b> should notice repetitive actions in computation and look for more shortcut methods. For example, students may use the distributive property as a strategy for using products they know to solve products that they don't know. For example, if students are asked to find the product of $7 \times 8$ , they might decompose 7 into 5 and 2 and then multiply $5 \times 8$ and $2 \times 8$ to arrive at $40 + 16$ or 56. In addition, third graders continually evaluate their work by asking themselves, "Does this make sense?"



## Effective Mathematics Teaching Practices

**Establish mathematics goals to focus learning.** Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

**Implement tasks that promote reasoning and problem solving.** Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

**Use and connect mathematical representations.** Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

**Facilitate meaningful mathematical discourse.** Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

**Pose purposeful questions.** Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

**Build procedural fluency from conceptual understanding.** Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

**Support productive struggle in learning mathematics.** Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

**Elicit and use evidence of student thinking.** Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

## **5 Practices for Orchestrating Productive Mathematics Discussions**

Practice	Description/ Questions
1. Anticipating	<p>What strategies are students likely to use to approach or solve a challenging high-level mathematical task?</p> <p>How do you respond to the work that students are likely to produce?</p> <p>Which strategies from student work will be most useful in addressing the mathematical goals?</p>
2. Monitoring	<p>Paying attention to what and how students are thinking during the lesson.</p> <p>Students working in pairs or groups</p> <p>Listening to and making note of what students are discussing and the strategies they are using</p> <p>Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)</p>
3. Selecting	<p>This is the process of deciding the <i>what</i> and the <i>who</i> to focus on during the discussion.</p>
4. Sequencing	<p>What order will the solutions be shared with the class?</p>
5. Connecting	<p>Asking the questions that will make the mathematics explicit and understandable.</p> <p>Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.</p>

## MATH CENTERS/ WORKSTATIONS

*Math workstations* allow students to engage in authentic and meaningful hands-on learning. They often last for several weeks, giving students time to reinforce or extend their prior instruction. Before students have an opportunity to use the materials in a station, introduce them to the whole class, several times. Once they have an understanding of the concept, the materials are then added to the work stations.

### Station Organization and Management Sample

Teacher A has 12 containers labeled 1 to 12. The numbers correspond to the numbers on the rotation chart. She pairs students who can work well together, who have similar skills, and who need more practice on the same concepts or skills. Each day during math work stations, students use the center chart to see which box they will be using and who their partner will be. Everything they need for their station will be in their box. **Each station is differentiated.** If students need more practice and experience working on numbers 0 to 10, those will be the only numbers in their box. If they are ready to move on into the teens, then she will place higher number activities into the box for them to work with.



In the beginning there is a lot of prepping involved in gathering, creating, and organizing the work stations. However, once all of the initial work is complete, the stations are easy to manage. Many of her stations stay in rotation for three or four weeks to give students ample opportunity to master the skills and concepts.

Read *Math Work Stations* by Debbie Diller.

In her book, she leads you step-by-step through the process of implementing work stations.

**MATH WORKSTATION INFORMATION CARD**

**Math Workstation:** \_\_\_\_\_

**Time:**

**NJSLS:**

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

**Objective(s):** By the end of this task, I will be able to:

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

**Task(s):**

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

**Exit Ticket:**

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

# MATH WORKSTATION SCHEDULE

Week of: \_\_\_\_\_

DAY	Technology Lab	Problem Solving Lab	Fluency Lab	Math Journal	Small Group Instruction
Mon.	Group ____	Group ____	Group ____	Group ____	<b>BASED ON CURRENT OBSERVATIONAL DATA</b>
Tues.	Group ____	Group ____	Group ____	Group ____	
Wed.	Group ____	Group ____	Group ____	Group ____	
Thurs.	Group ____	Group ____	Group ____	Group ____	
Fri.	Group ____	Group ____	Group ____	Group ____	
	Group ____	Group ____	Group ____	Group ____	

## INSTRUCTIONAL GROUPING

	GROUP A		GROUP B
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	
	GROUP C		GROUP D
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	

### Third Grade PLD Rubric

<b>Got It</b>		<b>Not There Yet</b>		
Evidence shows that the student essentially has the target concept or big math idea.		Student shows evidence of a major misunderstanding, incorrect concepts or procedure, or a failure to engage in the task.		
<b>PLD Level 5: 100% Distinguished command</b>	<b>PLD Level 4: 89% Strong Command</b>	<b>PLD Level 3: 79% Moderate Command</b>	<b>PLD Level 2: 69% Partial Command</b>	<b>PLD Level 1: 59% Little Command</b>
<p>Student work shows <b>distinct levels of understanding</b> of the mathematics.</p> <p>Student <b>constructs</b> and <b>communicates</b> a <b>complete response</b> based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> <li>• Tools: <ul style="list-style-type: none"> <li>○ Manipulatives</li> <li>○ Five Frame</li> <li>○ Ten Frame</li> <li>○ Number Line</li> <li>○ Part-Part-Whole Model</li> </ul> </li> <li>• Strategies: <ul style="list-style-type: none"> <li>○ Drawings</li> <li>○ Counting All</li> <li>○ Count On/Back</li> <li>○ Skip Counting</li> <li>○ Making Ten</li> <li>○ Decomposing Number</li> </ul> </li> <li>• Precise use of math vocabulary</li> </ul> <p>Response includes an <b>efficient and logical progression</b> of mathematical reasoning and understanding.</p>	<p>Student work shows <b>strong levels of understanding</b> of the mathematics.</p> <p>Student <b>constructs</b> and <b>communicates</b> a <b>complete response</b> based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> <li>• Tools: <ul style="list-style-type: none"> <li>○ Manipulatives</li> <li>○ Five Frame</li> <li>○ Ten Frame</li> <li>○ Number Line</li> <li>○ Part-Part-Whole Model</li> </ul> </li> <li>• Strategies: <ul style="list-style-type: none"> <li>○ Drawings</li> <li>○ Counting All</li> <li>○ Count On/Back</li> <li>○ Skip Counting</li> <li>○ Making Ten</li> <li>○ Decomposing Number</li> </ul> </li> <li>• Precise use of math vocabulary</li> </ul> <p>Response includes a <b>logical progression</b> of mathematical reasoning and understanding.</p>	<p>Student work shows <b>moderate levels of understanding</b> of the mathematics.</p> <p>Student <b>constructs</b> and <b>communicates</b> a <b>complete response</b> based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> <li>• Tools: <ul style="list-style-type: none"> <li>○ Manipulatives</li> <li>○ Five Frame</li> <li>○ Ten Frame</li> <li>○ Number Line</li> <li>○ Part-Part-Whole Model</li> </ul> </li> <li>• Strategies: <ul style="list-style-type: none"> <li>○ Drawings</li> <li>○ Counting All</li> <li>○ Count On/Back</li> <li>○ Skip Counting</li> <li>○ Making Ten</li> <li>○ Decomposing Number</li> </ul> </li> <li>• Precise use of math vocabulary</li> </ul> <p>Response includes a <b>logical but incomplete progression</b> of mathematical reasoning and understanding. Contains <b>minor errors</b>.</p>	<p>Student work shows <b>partial understanding</b> of the mathematics.</p> <p>Student <b>constructs</b> and <b>communicates</b> an <b>incomplete response</b> based on student's attempts of explanations/ reasoning using the:</p> <ul style="list-style-type: none"> <li>• Tools: <ul style="list-style-type: none"> <li>○ Manipulatives</li> <li>○ Five Frame</li> <li>○ Ten Frame</li> <li>○ Number Line</li> <li>○ Part-Part-Whole Model</li> </ul> </li> <li>• Strategies: <ul style="list-style-type: none"> <li>○ Drawings</li> <li>○ Counting All</li> <li>○ Count On/Back</li> <li>○ Skip Counting</li> <li>○ Making Ten</li> <li>○ Decomposing Number</li> </ul> </li> <li>• Precise use of math vocabulary</li> </ul> <p>Response includes an <b>incomplete or illogical progression</b> of mathematical reasoning and understanding.</p>	<p>Student work shows <b>little understanding</b> of the mathematics.</p> <p>Student <b>attempts to construct</b> and <b>communicates</b> a response using the:</p> <ul style="list-style-type: none"> <li>• Tools: <ul style="list-style-type: none"> <li>○ Manipulatives</li> <li>○ Five Frame</li> <li>○ Ten Frame</li> <li>○ Number Line</li> <li>○ Part-Part-Whole Model</li> </ul> </li> <li>• Strategies: <ul style="list-style-type: none"> <li>○ Drawings</li> <li>○ Counting All</li> <li>○ Count On/Back</li> <li>○ Skip Counting</li> <li>○ Making Ten</li> <li>○ Decomposing Number</li> </ul> </li> <li>• Precise use of math vocabulary</li> </ul> <p>Response includes <b>limited evidence of the progression</b> of mathematical reasoning and understanding.</p>
<b>5 points</b>	<b>4 points</b>	<b>3 points</b>	<b>2 points</b>	<b>1 point</b>

## DATA DRIVEN INSTRUCTION

Formative assessments inform instructional decisions. Taking inventories and assessments, observing reading and writing behaviors, studying work samples and listening to student talk are essential components of gathering data. When we take notes, ask questions in a student conference, lean in while a student is working or utilize a more formal assessment we are gathering data. Learning how to take the data and record it in a meaningful way is the beginning of the cycle.

Analysis of the data is an important step in the process. What is this data telling us? We must look for patterns, as well as compare the notes we have taken with work samples and other assessments. We need to decide what are the strengths and needs of individuals, small groups of students and the entire class. Sometimes it helps to work with others at your grade level to analyze the data.

Once we have analyzed our data and created our findings, it is time to make informed instructional decisions. These decisions are guided by the following questions:

- What mathematical practice(s) and strategies will I utilize to teach to these needs?
- What sort of grouping will allow for the best opportunity for the students to learn what it is I see as a need?
- Will I teach these strategies to the whole class, in a small guided group or in an individual conference?
- Which method and grouping will be the most effective and efficient? What specific objective(s) will I be teaching?

Answering these questions will help inform instructional decisions and will influence lesson planning.

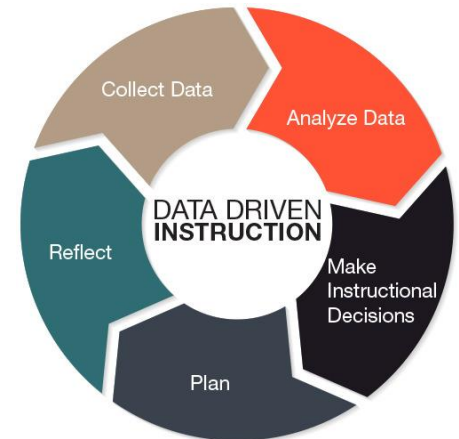
Then we create our instructional plan for the unit/month/week/day and specific lessons.

It's important now to reflect on what you have taught.

Did you observe evidence of student learning through your checks for understanding, and through direct application in student work?

What did you hear and see students doing in their reading and writing?

Now it is time to begin the analysis again.



**Data Analysis Form**

School: \_\_\_\_\_

Teacher: \_\_\_\_\_

Date: \_\_\_\_\_

Assessment: \_\_\_\_\_

NJSLS: \_\_\_\_\_

GROUPS (STUDENT INITIALS)	SUPPORT PLAN	PROGRESS
MASTERED (86% - 100%) (PLD 4/5):		
DEVELOPING (67% - 85%) (PLD 3):		
INSECURE (51%-65%) (PLD 2):		
BEGINNING (0%-50%) (PLD 1):		



## MATH PORTFOLIO EXPECTATIONS

The **Student Assessment Portfolios for Mathematics** are used as a means of documenting and evaluating students' academic growth and development over time and in relation to the CCSS-M. The September task entry(-ies) should reflect the prior year content and *can serve* as an additional baseline measure.

All tasks contained within the **Student Assessment Portfolios** should be aligned to NJSL and be “practice forward” (closely aligned to the Standards for Mathematical Practice).

Four (4) or more additional tasks will be included in the **Student Assessment Portfolios** for Student Reflection and will be labeled as such.

### K-2 GENERAL PORTFOLIO EXPECTATIONS:

- Tasks contained within the Student Assessment Portfolios are “practice forward” and denoted as “Individual”, “Partner/Group”, and “Individual w/Opportunity for Student Interviews<sup>1</sup>”.
- Each Student Assessment Portfolio should contain a “Task Log” that documents all tasks, standards, and rubric scores aligned to the performance level descriptors (PLDs).
- Student work should be attached to a completed rubric; with appropriate teacher feedback on student work.
- Students will have multiple opportunities to revisit certain standards. Teachers will capture each additional opportunity “as a new and separate score” in the task log.
- A 2-pocket folder for each Student Assessment Portfolio is *recommended*.
- All Student Assessment Portfolio entries should be scored and recorded as an Authentic Assessment grade (25%)<sup>2</sup>.
- All Student Assessment Portfolios must be clearly labeled, maintained for all students, inclusive of constructive teacher and student feedback and accessible for review.

## GRADES K-2

### Student Portfolio Review

Provide students the opportunity to review and evaluate their portfolio at various points throughout the year; celebrating their progress and possibly setting goals for future growth. During this process, students should retain ALL of their current artifacts in their Mathematics Portfolio

**Authentic Assessment**– Strips of Paper

Measure the strips of paper to the nearest  $\frac{1}{2}$  inch. Use the data to create a line plot. Be sure to label and include a title.

Name two facts that describe the data on your line plot.

- 1.) \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_
  
- 2.) \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

[Redacted]

[Redacted]

[Redacted]

[Redacted]

[Redacted]

[Redacted]

[Redacted]

[Redacted]

[Redacted]

[Redacted]

**3.MD.4:** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.

No Command	Partial Accomplishment	Substantial Accomplishment	Complete Mastery
All is incorrect	<p>Students who demonstrate partial accomplishment may measure the strips accurately, but may not be able to complete the line plot correctly.</p> <p>OR</p> <p>Students might have difficulty measuring the strips accurately, which would result in incorrect results on the line plot.</p>	<p>Students who demonstrate substantial accomplishment accurately measure the lengths of all of the strips and correctly use one x for each measurement on the line plot. But they might have difficulty/need assistance stating two facts about their line plot.</p>	<p>Students who demonstrate complete mastery accurately measure the lengths of all of the strips and correctly use one x for each measurement on the line plot. Students should also be able to write two facts about their line plot.</p>

## Resources

### **Engage NY**

[http://www.engageny.org/video-library?f\[0\]=im\\_field\\_subject%3A19](http://www.engageny.org/video-library?f[0]=im_field_subject%3A19)

### **Common Core Tools**

<http://commoncoretools.me/>

<http://www.ccsstoolbox.com/>

<http://www.achievethecore.org/steal-these-tools>

### **Achieve the Core**

<http://achievethecore.org/dashboard/300/search/6/1/0/1/2/3/4/5/6/7/8/9/10/11/12>

### **Manipulatives**

<http://nlvm.usu.edu/en/nav/vlibrary.html>

<http://www.explorelearning.com/index.cfm?method=cResource.dspBrowseCorrelations&v=s&id=USA-000>

<http://www.thinkingblocks.com/>

**Illustrative Math Project** :<http://illustrativemathematics.org/standards/k8>

**Inside Mathematics:** <http://www.insidemathematics.org/index.php/tools-for-teachers>

**Sample Balance Math Tasks:** <http://www.nottingham.ac.uk/~ttzedweb/MARS/tasks/>

**Georgia Department of Education:**<https://www.georgiastandards.org/Common-Core/Pages/Math-K-5.aspx>

**Gates Foundations Tasks:**<http://www.gatesfoundation.org/college-ready-education/Documents/supporting-instruction-cards-math.pdf>

### **Minnesota STEM Teachers' Center:**

<http://www.scimathmn.org/stemtc/frameworks/721-proportional-relationships>

**Singapore Math Tests K-12:** <http://www.misskoh.com>

**Mobymax.com:** <http://www.mobymax.com>

# 21st Century Career Ready Practices

- CRP1. Act as a responsible and contributing citizen and employee.
- CRP2. Apply appropriate academic and technical skills.
- CRP3. Attend to personal health and financial well-being.
- CRP4. Communicate clearly and effectively and with reason.
- CRP5. Consider the environmental, social and economic impacts of decisions.
- CRP6. Demonstrate creativity and innovation.
- CRP7. Employ valid and reliable research strategies.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP9. Model integrity, ethical leadership and effective management.
- CRP10. Plan education and career paths aligned to personal goals.
- CRP11. Use technology to enhance productivity.
- CRP12. Work productively in teams while using cultural global competence.

For additional details see **21<sup>st</sup> Century Career Ready Practices** .

## References

“Eureka Math” *Great Minds*. 2018 < <https://greatminds.org/account/products>>